SCIENCEFORECAST
A digital science library

Research Article
Published: 19 Mar, 2018

SF Journal of Aviation and Aeronautical Science

Altitude Controlling and Trajectory Tracking of Quadrotor UAV by using Model Predictive Control

Shafiq M, Ali ZA*, Ahmed MA and Farooq F

Electronic Engineering Department, Sir Syed University of Engineering and Technology, Karachi, Pakistan

Abstract

In this paper, Model Predictive Controller (MPC) scheme is proposed to stabilize the desired altitude and attitude of the Quad-rotor-Unmanned Aerial Vehicle (Q-UAV). The proposed control strategy uses control input as a reference which is utilized to track the referred trajectory, such that the linear and angular velocities which are used to derive the model of the Q-UAV. MPC is used as a main controller to control the dynamics of Q-UAV, while the nonlinear behavior and stability of the under-actuated Q-UAV is dealt by Extended Kalman Filter (EKF), which is also utilized to predict the desired location of UAV. The proposed control strategy is verified by using multiple simulations in Simulink MATLAB. The efficiency of the proposed controller is compared with Proportional Integral Derivative (PID) controller. It shows that it exhibits minimum steady state errors and fast error convergence in the presence of model uncertainties.

Keywords: Model Predictive Control (MPC); Extended Kalman Filter (EKF); Quad-rotor Unmanned Aerial Vehicle (Q-UAV)

Introduction

Unmanned Aerial Vehicle (UAV) has realized rapid progress, mainly due to the capability of the UAV to efficiently bring out a number of applications at very least cost and with minimum risk of human resources. At present, different type of missions including search and rescue mission, wild fire investigation, nuclear reactors observations, power plants checkups, farming services, recording and photographing, naval operations, battle damage assessments, border interdiction preventions, and law enforcement are the core applications of UAV [1-3].

The above mentioned detail set of possible applications have modern demands in the areas of control and direction finding techniques. In order to design unmanned systems, which is capable of operating in unadorned environments and handling with complex missions, better controlling techniques are needed to overcome the error ignored by the other methods [4]. All types of aerial systems, including manned and unmanned aerial vehicle, rotorcraft are the best solution of any mission due to its ability of Vertical Takeoff and Landing (VTOL) as well as hostile maneuverability [5,6]. Moreover, the control and design problems are associated with the rotorcrafts plays an important role in order to fly efficiently and autonomously [7]. The advanced watercraft utilizes aggressive dynamics due to Low inertia moments as well as aerodynamic effects on their flights. State estimations and controller implementation with efficient rates are necessary requirements, but the low cost system with on board sensors and actuators are very noisy and give drifts when operate in the air, therefore the control problems are difficult to manage and complexity increased [8,9].

Previously, many control algorithms have been used to control the altitude and trajectory tracking of rotorcrafts. In [10] Robust Proportional Integral Derivative (PID) controller is used to control the altitude of UAV. The gain scheduling based PID controller is utilized for the fault control of Quad-Rotor UAV in [11]. In reference [12] the hover control of unmanned helicopter is done by using Enhanced-Linear Quadratic Regulators (E-LQR). In [13] feedback linearization versus adaptive sliding mode control scheme is used to stabilize the dynamics of quad-rotor UAV. However, to control the nonlinear behavior of a Quad-rotor mini UAV by using the feedback-linearization technique is used. The dynamic feedback controller, which is used to control the Euler angles and wind disturbance estimation of Quad-rotor is done in [14]. Moreover, in this research article, our main objective is to stabilize the desired altitude, attitude of UAV.

The proposed control algorithm uses EKF to linearize the nonlinear behavior of the system as well as it is also used to predict the next state of UAV. The core contributions of this article are as

****Correspondence:***
Zain Anwar Ali, Electronic Engineering Department, Sir Syed University of Engineering and Technology, Karachi, Pakistan.
E-mail: *zainanwar86@hotmail.com*

Received Date: *13 Feb 2018*
Accepted Date: *14 Mar 2018*
Published Date: *19 Mar 2018*

Citation: *Shafiq M, Ali ZA, Ahmed MA, Farooq F. Altitude Controlling and Trajectory Tracking of Quadrotor UAV by using Model Predictive Control. SF J Aviation Aeronaut Sci. 2018; 1(1): 1009.*

ISSN 2643-8119

follows, (a) to control the position as well as tracking the trajectory of the Quad-rotor UAV using EKF Algorithm in MPC scheme, (b) the proposed control scheme contain angular velocity constraints as an input control commands, which is quite effective, accurate and observed from real world scenario, (c) the efficiency of the designed control algorithm is compared with classical PID controller and it shows better robustness and quick convergence.

This paper has a great impact on the research work related to the altitude controlling and trajectory tracking of UAV. The first task is to reach at the desired altitude and after that the UAV must maintain its hovering state than it will move along the x and y axis accordingly. In other words, we can say that the altitude holds the position (z fixed) and varying its remaining axis to obey the referred trajectory with minimum margins of errors.

The rest of the manuscript is organized as follows. Section 2 defines the Preliminaries and the model of the Quad - rotor UAV. Control algorithm of the Quad - rotor UAV is defined in section 3. The overall simulations and results of this research article by using two different objectives are shown in section 4. Section 5 concludes the whole research article.

System Preliminaries

In the following divisions mathematical model of the UAV as well as a physical model with its linearization is defined.

System model

In the system model, mathematical approach is obtained through its rotational matrix including translational and rotational subsystem of the proposed UAV. The circumference of the circle (earth) in which UAV is flying calculate to find the exact position of the UAV through global coordinate system (also called earth coordinate system) by the formula,

$$Sc = \lambda Rc \tag{1}$$

Where "λ" is the turning angle and "Rc" is the radius of the circular earth (Globe). With reference to the equation (02) and (03), longitudinal position as well as lateral of the UAV is calculated by using the Earth's surface [15].

$$\begin{cases} X_p = \lambda_{lat.}\left(\dfrac{\pi}{180}\right) * R_{earth} \\ Y_p = \lambda_{lat.}\cos\left(\lambda_{lat.} * \dfrac{\pi}{180}\right) * R_{earth} \end{cases} \tag{2}$$

To find the altitude rectangular coordinate axes (x, y and z) Euler angles (φ, θ, ψ) are utilized. These axes are now termed as Roll, Pitch and Yaw respectively [16]. The Rotation matrix is defined as,

$$\begin{cases} R_\varphi^x R_\theta^y R_\psi^y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ R_\varphi^x R_\theta^y R_\psi^z = \begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\psi\sin\psi & \sin\varphi\sin\theta\sin\psi - \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\cos\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\theta\cos\psi & \cos\psi\cos\theta \end{pmatrix} \end{cases} \tag{3}$$

The proposed model of the quad-rotor UAV based on three motors fixed in the body shaped as square, force and torque is written as,

$$\begin{cases} f_i = ktu_i^2 \rightarrow ktu_i\left|u_i\right| \\ \tau_i = k\tau u_i^2 \rightarrow k\tau u_i\left|u_i\right| \end{cases} \tag{4}$$

Where i=1,2,3 for all three rotors of the UAV.

Similarly, external forces& mass of the model can be written as,

$$\begin{cases} F_{ext.} = \begin{pmatrix} F_x \\ Fy \\ F_z \end{pmatrix} = \begin{pmatrix} 0 \\ kt\left(u_1\left|u_1\right|\sin u_4\right) \\ -kt\left(u_1\left|u_1\right|\cos u_4 + u_2\left|u_2\right| + u_3\left|u_3\right|\right) \end{pmatrix} \\ M_{F_{ext.}} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} \left(\dfrac{\sqrt{3}}{2}\right) l * kt\left(u_2\left|u_2\right| - u_3\left|u_3\right|\right) \\ 0.5 * l * kt\left(u_2\left|u_2\right| + u_3\left|u_3\right|\right) - l * kt\left(u_1\left|u_1\right|\cos u_4\right) + k\tau\left(u_1\left|u_1\right|\sin u_4\right) \\ -l * kt\left(u_1\left|u_1\right|\sin u_4\right) - k\tau\left(u_1\left|u_1\right|\cos u_4\right) + u_2\left|u_2\right| + u_3\left|u_3\right| \end{pmatrix} \end{cases} \tag{5}$$

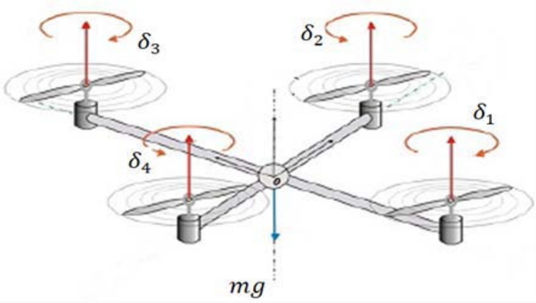

Figure 1: The physical model of quad-rotor UAV.

Translational system dynamics are,

$$\begin{cases} \dot{u} = (1/m)\left(F_x\cos\theta\cos\psi + F_y\cos\theta\sin\psi - F_z\sin\theta\right) \\ \dot{v} = (1/m)\left(F_x\left(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi\right) + F_y\left(\sin\varphi\sin\theta\sin\psi + \cos\theta\cos\psi\right) + F_z\left(\sin\varphi\cos\theta\right)\right) \\ \dot{w} = (1/m)\left(F_x\left(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi\right) + F_y\left(\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi\right) + F_z\left(\cos\varphi\cos\theta\right)\right) \end{cases} \tag{6}$$

Rotational system dynamics are,

$$\begin{cases} \dot{p} = \varphi - \psi\sin\theta \\ \dot{q} = \theta\cos\varphi + \psi\sin\varphi\cos\theta \\ \dot{r} = \psi\cos\varphi\cos\theta - \kappa\sin\varphi \end{cases} \tag{7}$$

The above mentioned equations from (04 - 07) is taken from [17 -21]. The constants of the proposed UAV system has shown in Table 1 where I is the inertia components in all three axes, l is the arm length of the Q-UAV, m is the mass, b is the thrust coefficient and d is drag coefficient [22]. In Figure 1, the physical model of the Q-UAV is presented where the four propellers are synchronized with the main controller where two opposite wings rotate in the same direction [23].

Physical model and its linearization

The inputs of all three rotors and rotational subsystems of the system model of the UAV is used to derive the system which has previously been used in [16], in which the state space vector is "X", the input and output vector are "U" and "Y".

$$\begin{cases} \dot{X} = f(X,U) \\ \dot{Y} = h(X) \end{cases} \tag{8}$$

Following is the state space model of the system,

$$\begin{cases} X = [\dot{p},\dot{q},\dot{r},u_1,u_2,u_3,u_4] \\ U = [u_1,u_2,u_3,u_4] \end{cases} \tag{9}$$

$$\begin{cases} u_1 = b\left(\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2\right) \\ u_2 = b\left(-\delta_2^2 + \delta_4^2\right) \\ u_3 = b\left(\delta_1^2 - \delta_3^2\right) \\ u_1 = d\left(-\delta_1^2 + \delta_2^2 - \delta_3^2 + \delta_4^2\right) \end{cases} \tag{10}$$

$$Y = [p,q,r] \tag{11}$$

At the present, rotation rates of the system as well as its electrical motors input are formed linearization of the physical model. The steady state response of the system is used to linearize the nonlinear behavior of the system. After that this response will be equalized to zero when the steady state equation becomes equal to the equilibrium of the system. Now the UAV must be linearize just about the equilibrium points X_0', U_0'.

$$f\left(X_0', U_0'\right)0 \tag{12}$$

Trim points conditions are performing linearization of this nonlinear system which is basically the operating point of the UAV under input torque on the rotors of the UAV.

$$A=\begin{bmatrix} -10^{-11} & 0 & 0 & 0 & \left(-\frac{1}{I_x}\right)\left(\sqrt{3}*l*kt*u_2\right) & \left(-\frac{1}{I_y}\right)\left(\sqrt{3}*l*kt*u_3\right) & 0 \\ 0 & -10^{-11} & 0 & \left(\frac{1}{I_y}\right)(2*u_1)(kt*\sin u_4-l*kt*\cos u_4) & \left(\frac{1}{I_y}\right)(l*kt*u_2) & \left(\frac{1}{I_y}\right)(l*kt*u_3) & \left(\frac{1}{I_y}\right)(u_1|u_3|)(kt*k\tau*\sin u_4+k\tau*\cos u_4) \\ 0 & 0 & -10^{-11} & \left(-\frac{1}{I_z}\right)(2*u_1)(l*kt*\sin u_4+k\tau*\cos u_4) & \left(-\frac{1}{I_z}\right)(2*kt*u_2) & \left(-\frac{1}{I_z}\right)(2*kt*u_3) & \left(\frac{1}{I_z}\right)(u_1|u_1|)(k\tau*\sin u_4-l*kt*\cos u_4) \\ 0 & 0 & 0 & -10^{-9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10^{-9} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -10^{-9} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ B_{4.1} & 0 & 0 & 0 \\ 0 & B_{5.2} & 0 & 0 \\ 0 & 0 & B_{6.31} & 0 \\ 0 & 0 & 0 & B_{7.4} \end{bmatrix} \qquad (13)$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now $B_{4.1}$, $B_{5.2}$, $B_{6.3}$ & $B_{7.4}$ are the constants.

Control Algorithm

Model Predictive Control (MPC) is a popular control algorithm because it deals with the constraints. Using all constraints, the computation of the control signals needs to be real, time at each sampling time. Previously in MPC the forward shift operators were the key model for prediction of the trajectory control. However, the complex system needs wide control horizon which will improve the number of factors involve for real time computation. The above condition does not apply to the small scale systems where computation is limited. Additionally, in the past, the future control trajectory was predicted by the Laguerre method which is basically responsible for the decrement of the number of factors of real time computation therefore mathematics decreased.

Following equations represent the MPC algorithm where the controlling has shown in discrete time.

$$\begin{cases} x_d(i+1)=A_d x_d(i)+B_d u(i) \\ y(i+1)=C_d x_d(i) \\ \Delta x_d(i+1)=x_d(i+1)-x_d(i) \\ \Delta u(i)=u(i)-u(i-1) \end{cases} \qquad (14)$$

$$\begin{bmatrix} \Delta x_d(i+1) \\ y(i+1) \end{bmatrix}=\begin{bmatrix} A_d & O_{d\times 1} \\ C_d A_d & I_{n\times n} \end{bmatrix}\begin{bmatrix} \Delta x_d(i) \\ y(i) \end{bmatrix}+\begin{bmatrix} B_d \\ C_d B_d \end{bmatrix}\Delta u(i) \qquad (15)$$

Table 1: Constants of UAV system.

Parameter Symbol	Parameter Description
I_x	x-xaxis inertia component
I_y	y-xaxis inertia component
I_z	z-xaxis inertia component
l	Length of the quadrotor arm
m	Mass of quadrotor
b	Thrust co-efficient
d	Drag co-efficient

Table 2: The controller parameters of the system.

Parameter	Value	Unit
m	0.8	Kg
l	0.3	M
I_x	15.67 x 10^{-3}	Kg m^2
I_y	15.67 x 10^{-3}	Kg m^2
I_z	28.346 x 10^{-3}	Kg m^2
b	192.3208 x 10^{-7}	Ns^2
d	4.003 x 10^{-7}	Nms^2

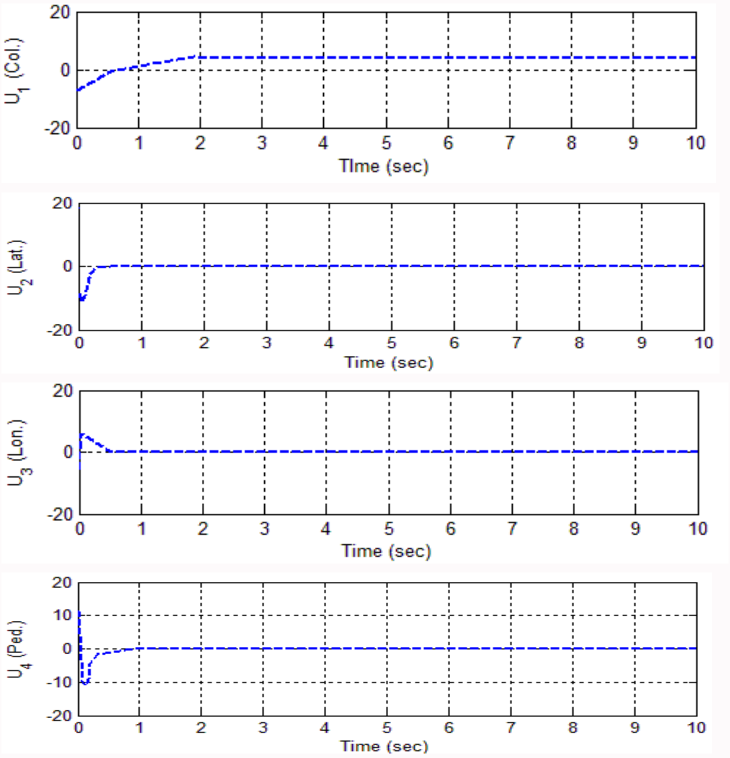

Figure 2: (a), (b), (c) and (d) Controller inputs of the Quad-rotor.

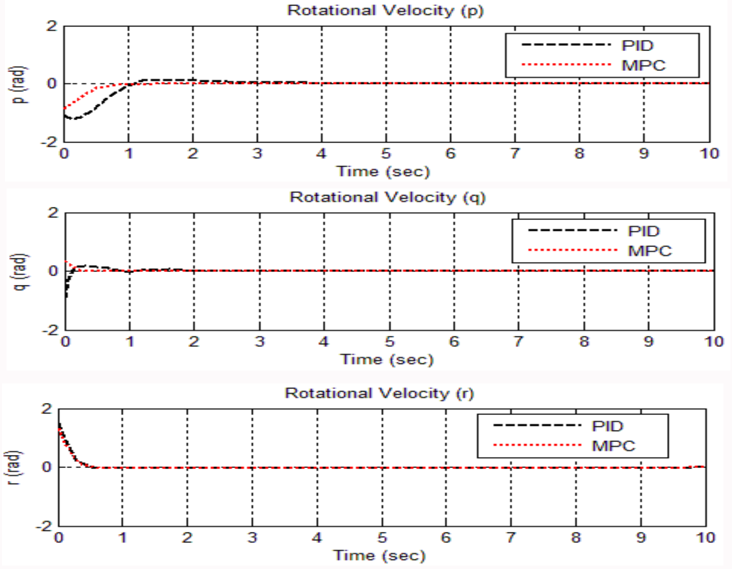

Figure 3: (a), (b) and (c) Rotational Velocity components of the Quad-rotor.

The total system dynamics of the quad-rotor are,

$$\begin{bmatrix} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{bmatrix} \tag{16}$$

The above equations from (14 - 16) of the control algorithm are taken from [23]. At present, most process industry control systems are usually based on the Model Predictive Control approach followed by the steps of digital techniques. Initially the controller receives the modern plant output at the instant of time. By using this, the future system output is predicted by the prediction horizon “Mp“. Afterwards the controller locates the better control signal with respect to the control horizon “Mc” using the method of optimization. Moreover, the continuous predictions of the future states depend upon the sample taken at the start of the process.

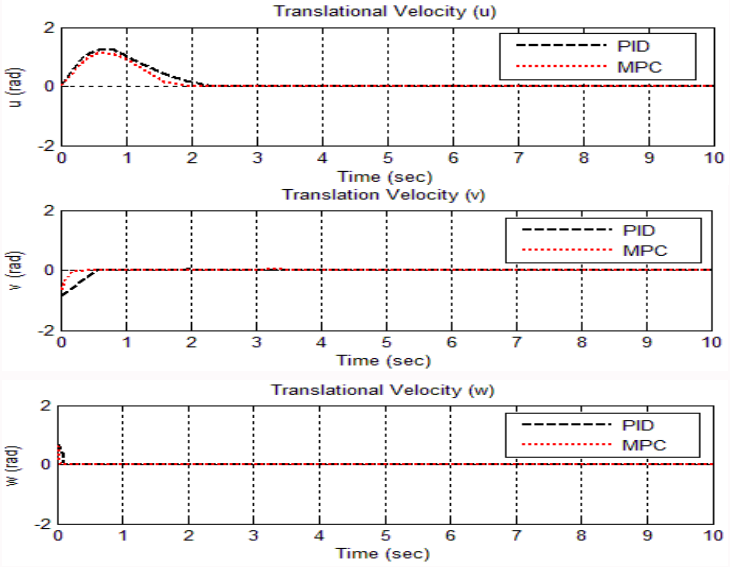

Figure 4: (a), (b) and (c) Translational Velocity components of the Quad-rotor.

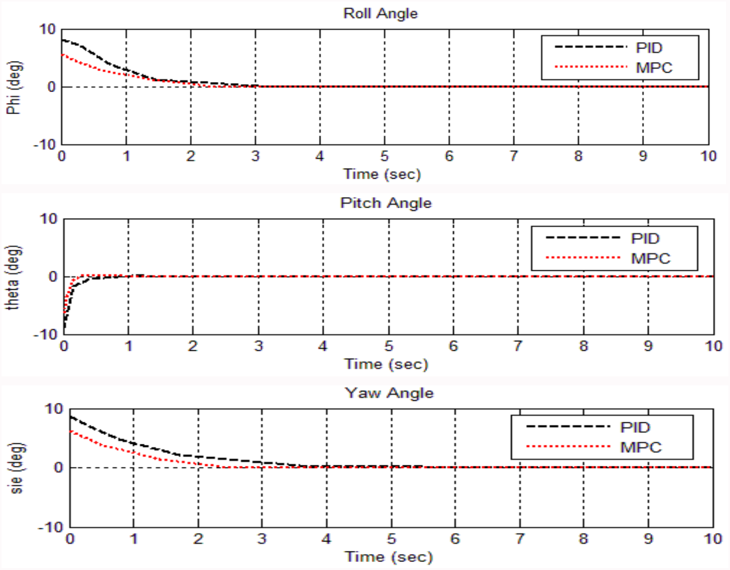

Figure 5: (a), (b) and (c) Angular Response of the Quad-rotor.

Simulation and Results

In the following sections different simulations have been performed to obtain desired results for the dynamics of the Q-UAV [24]. The controller parameters of UAV system are defined in Table 2. In the initial phase, three basic approaches are utilized as robustness, effectiveness and strength followed by Extended Kalman Filter (EKF). On the other hand different inputs are applied to stabilize the controller and verify the desired result by using MATLAB Simulink. In the first simulation diagram the stabilization of the altitude and trajectory tracking of the Q-UAV has been presented. Moreover, the results came after the three basic approaches as robustness, effectiveness and durability. Finally the desired results are achieved with marginal errors and give better compared with the other techniques.

The altitude of the Q-UAV in the simulations are (z=0) m and (0, 0, 0) deg. The altitude and attitude angles of the proposed controller are utilized in the above simulations are: z=4m, ϕ=0, θ=0 deg, ψ=0 deg which are shown in Figures 2 and 3respectively. For best rates of convergence and minimum steady state error, the proposed controller is tuned to the following specifications shown with the simulations. The translational and rotational velocity components are shown in the following simulation diagrams, as well as the behavior of the response is related with respect to the altitude and attitude angles. All the components of the velocity subcomponents converge to zero with the suitable interval of time that will lead to linear behavior.

All the controller inputs (u1, u2, u3 and u4) converge to zero, respectively. Furthermore firstly vibrations are present in the proposed Extended Kalman Filter based Model Predictive Control (MPC) controllers as compared to other controllers. In the first simulation Figures

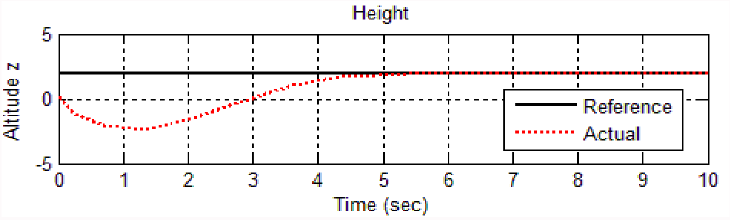

Figure 6: Altitude Response of the Quad-rotor.

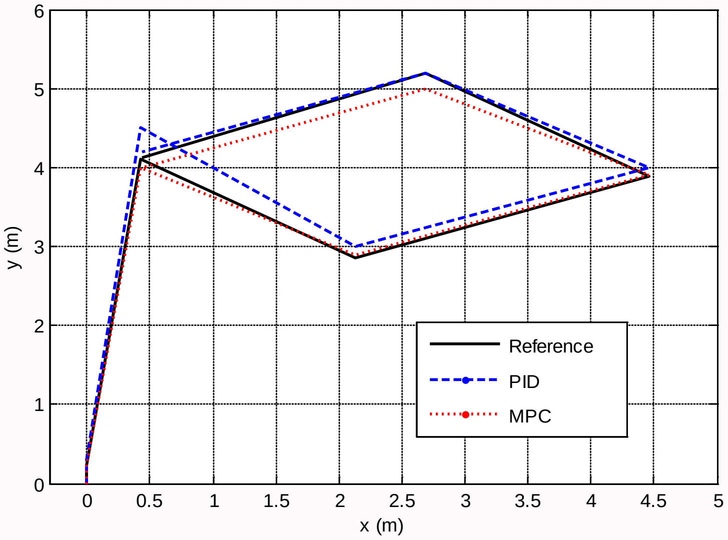

Figure 7: 2D diagram presents the referred trajectory of proposed Q-UAV using Extended Kalman Filter based Model Predicted Control Algorithm.

2(a), (b), (c) and (d),all the controller inputs converges to zero with the average delay of 0.5sec. Initially the u1 takes the large time for converging to zero, but the other inputs take under 0.5sec delay to achieve its goals. In Figure 3(a), (b) and (c) rotational velocities of the proposed MPC controller are compared with the existing PID controller method and the results shows that the MPC gives more stability over PID for the required altitude and preferred trajectory. In Figure 4(a), (b) and (c) translational velocities of the proposed MPC controller are compared with the PID controller method and the results shows that the MPC gives more stability over PID for the required altitude and preferred trajectory. In Figure 5(a), (b) and (c) angular response of the proposed MPC controller are compared with the PID controller techniques and the results shows that the it gives better response time to stable over PID. In Figure 6, the quad-rotor initially starts from a reference platform in the downward direction and then within 4 seconds quad-rotor achieves its fixed altitude for hovering. After 5 seconds the Q-UAV is completely stable and follows the squared trajectory.

The tracking errors of the squared trajectory of UAV along rectangular coordinates are converging to zero initially, but when the referred movement and orientation of the system changes, it generates errors. These errors identified and removed by above proposed methods. The 2D trajectory model of the proposed scheme gives improved steady state as well as transient response using Extended Kalman Filter based MPC control algorithm over traditional PID control as shown in Figure 7. The controlling parameters including Euler angles, control torques and velocity components with marginally undershoots and overshoots are stabilized (converges to zero) within a second.

The comparison of the proposed MPC controller with PID controller gives more accuracy to follow the referred squared trajectory of the quad-rotor over the span of 0.5 Sec and the followed coordinates of the square are closed enough to stable the system for hovering with a fixed altitude of 5m with respect to the platform.

Following are the equations of the required (squared) trajectory with our proposed MPC method and it gives an effective performance with fewer errors, Where tr = total thrust, Ød = Euler angles and θd =Attitude angles can be presented as,

$$\begin{cases} u_x(t) = t_r\left(\sin\theta_d \cos\psi_d \cos\phi_d + \sin\psi_d \sin\phi_d\right) \\ u_y(t) = t_r\left(\sin\theta_d \sin\psi_d \cos\phi_d - \cos\psi_d \sin\phi_d\right) \\ u_z(t) = t_r\left(\cos\theta_d \cos\phi_d\right) \end{cases} \tag{17}$$

$$\begin{cases} t_r = \left(u_x(t)^2 + u_y(t)^2 + u_z(t)^2\right)^{1/2} \\ \phi_d = \sin^{-1}\left(\dfrac{u_x(t)\sin\psi_d - u_y(t)\cos\psi_d}{t_r}\right) \\ \theta_d = \tan^{-1}\left(\dfrac{u_x(t)\cos\psi_d + u_y(t)\sin\psi_d}{u_z(t)}\right) \end{cases} \tag{18}$$

The above equations from (17 - 18) of the trajectory are taken from [25].

Conclusion

In this paper, Extended Kalman Filter based Model Predictive Control (MPC) algorithm is proposed for the altitude and trajectory tracking of the quad rotor Unmanned Aerial Vehicle (Q-UAV) for the estimated position and attitude tracking. The suggested Extended Kalman Filter based MPC control algorithm is verified by the two different simulation approaches using MATLAB Simulink. In the first simulation method, the proposed controller is tested with appropriate inputs to check the stability of the system and then tracking the squared trajectory of the system. On the other hand, the output of the MPC based proposed system produce much better stability compared to PID scheme and the system becomes more efficient for the desired results.

References

1. Andreas, Peter. "Redrawing the line: Borders and security in the twenty-first century." International security. 2003; 28: 78-111.
2. Bernard, Markus, Konstantin Kondak, Ivan Maza, and Anibal Ollero. "Autonomous transportation and deployment with aerial robots for search and rescue missions." Journal of Field Robotics. 2011; 28: 914-931.
3. Birk, Andreas, Burkhard Wiggerich, HeikoBülow, Max Pfingsthorn, and SörenSchwertfeger. "Safety, security, and rescue missions with an unmanned aerial vehicle (uav)." Journal of Intelligent & Robotic Systems. 2011; 64: 57-76.
4. Deng, Yueyue. "Task Allocation and Path Planning for." BID. 2001; 105: 1.
5. Gupta, Suraj G, Mangesh M Ghonge, and PM Jawandhiya. "Review of unmanned aircraft system (UAS)." International Journal of Advanced Research in Computer Engineering & Technology (IJARCET). 2013; 2: pp-1646.
6. Lozano, Rogelio, ed. Unmanned aerial vehicles: Embedded control. John Wiley & Sons, 2013.
7. Schafroth, Dario, Samir Bouabdallah, Christian Bermes, and Roland Siegwart. "From the test benches to the first prototype of the muFly micro helicopter." Journal of Intelligent and Robotic Systems. 2009; 54: 245-260.
8. Litt, Jonathan S, Donald L Simon, Sanjay Garg, Ten-HueiGuo, Carolyn Mercer, Richard Millar, et al. "A survey of intelligent control and health management technologies for aircraft propulsion systems." JACIC. 2004; 1: 543-563.
9. Amidi, Omead. "An autonomous vision-guided helicopter." 1996.
10. Sangyam, Theerasak, Pined Laohapiengsak, Wonlop Chongcharoen, and Itthisek Nilkhamhang. "Path tracking of UAV using self-tuning PID controller based on fuzzy logic." In SICE Annual Conference. 2010: 1265-1269.
11. Sadeghzadeh, Iman, Ankit Mehta, Youmin Zhang, and Camille-Alain Rabbath. "Fault-tolerant trajectory tracking control of a quadrotor helicopter using gain-scheduled PID and model reference adaptive control." In Annual conference of the prognostics and health management society. 2011; 2: 34.
12. Jiang, Zhe, Jianda Han, Yuechao Wang, and Qi Song. "Enhanced LQR control for unmanned helicopter in hover." In Systems and Control in Aerospace and Astronautics, 2006. ISSCAA 2006. 1st International Symposium on. pp. 6-pp. IEEE, 2006.
13. Johnson, Eric N., and Suresh K. Kannan. "Adaptive trajectory control for autonomous helicopters." Journal of Guidance Control and Dynamics. 2005; 28: 524-538.
14. Lee, Daewon, H. Jin Kim, and Shankar Sastry. "Feedback linearization vs. adaptive sliding mode control for a quadrotor helicopter." International Journal of control, Automation and systems. 2009; 7: 419-428.
15. Siebert, Sebastian, and JochenTeizer. "Mobile 3D mapping for surveying earthwork projects using an Unmanned Aerial Vehicle (UAV) system." Automation in Construction. 2014; 41: 1-14.
16. Barsk, Karl-Johan. "Model predictive control of a tricopter." 2012: 90.
17. Ali, Zain Anwar, Daobo Wang, and Muhammad Aamir. "Fuzzy-Based hybrid control algorithm for the stabilization of a tri-rotor UAV." Sensors. 2016; 16: 652.
18. Ali, Zain Anwar, Daobo Wang, SuhaibMasroor, and M. ShafiqLoya. "Attitude and Altitude Control of Trirotor UAV by Using Adaptive Hybrid Controller." Journal of Control Science and Engineering. 2016: 2016.
19. Ali, Zain Anwar, Dao Bo Wang, Muhammad Aamir, and SuhaibMasroor. "MRAC base robust RST control scheme for the application of UAV." International Journal of Modelling, Identification and Control. 2017; 28: 232-244.
20. Ali, Zain Anwar, Daobo Wang, Muhammad Aamir, and SuhaibMasroor. "Trajectory Tracking of a Tri-Rotor Aerial Vehicle Using an MRAC-Based Robust Hybrid Control Algorithm." Aerospace. 2017; 4: 3.
21. Ali, Zain Anwar, Dao-Bo Wang, Rana Javed, and Ali Akbar. "Modeling & Controlling the Dynamics of Tri-rotor UAV Using Robust RST Controller with MRAC Adaptive Algorithm." International Journal of Control and Automation. 2016; 9: 61-76.
22. Khan, Hamid Saeed, and Muhammad Bilal Kadri. "Position control of quadrotor by embedded PID control with hardware in loop simulation." In Multi-Topic Conference (INMIC), 2014 IEEE 17th International. 2014: pp. 395-400. IEEE, 2014.
23. Chen, Xi, and Liuping Wang. "Cascaded model predictive control of a quadrotor UAV." In Control Conference (AUCC), 2013 3rd Australian, pp. 354-359. IEEE, 2013.
24. Khan, Hamid Saeed, and Muhammad Bilal Kadri. "Attitude and altitude control of quadrotor by discrete PID control and non-linear model predictive control." In Information and Communication Technologies (ICICT), 2015 International Conference on, pp. 1-11. IEEE, 2015.

25. Chengshun Yang, Zhong Yang, Xiaoning Huang, Shaobin Li, and Qiang Zhang, “Modeling and Robust Trajectory Tracking Control for a Novel Six-Rotor Unmanned Aerial Vehicle.” Mathematical Problems in Engineering. 2013; 2013: Article ID 673525.